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Study of some modelling and analysis of thermoelastic problem using fractional calculus

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Abstract

In the research proposal, we formulate fractional thermoelastic models from classical thermoelasticity. This work is aimed to simplify the theoretical frameworks by clarifying connections between existed models in the literature. To this end, novel modified fractional thermoelasticity models are established by directly extending classical thermoelasticity with the aids of new forms of generalized fractional derivatives, i.e. generalized Caputo, generalized Hilfer, Caputo-Fabrizio, Atangana–Baleanu, and Tempered-Caputo definitions. Also, we prove the uniqueness theorems for these models.

Theoretically, the considered models may be simplified into existed fractional theory. The dynamic problems and the governing equations of fractional coupled thermoelasticity based on integral transformations are analyzed and derived.

Numerically, the considered models have the capability of describing thermoelastic behaviors from various fractional models by the Laplace and Fourier transform methods. Moreover, we apply the state-space approach, Laplace transformation, and Fourier transformation to solve some problems of fractional generalized thermoelasticity.

Finally, we develop some generalized thermoelastic models such as Povstenko model (2011), Green-Nigdhi (GN) Model (1993) and Sherief model (2010) in the frame of mentioned fractional calculus.

1 Introduction

1.1 Fractional calculus and modeling

Fractional calculus is a new-old subject. The idea of fractional derivative goes back to more than three hundred years. It concerns the generalization of the concept of derivative to a non-integer order. Several kinds of fractional derivatives have been defined: the Riemann-Liouville, Weyl, Weyl-Liouville, Chen, Bessel, Marchaud, Grünwald-Letnikov, Dzerbashyan, Hadamard, etc...
The Riemann-Liouville derivative is the most commonly used especially by mathematicians. However, the initial conditions associated with this derivative are not physically meaningful and there no ways how measure them. Yet, stil some of these conditions have physical meaning , see the papers by N. Heymans and I. Podlubny [5] and Podlubny [8].
Physicists and engineers prefer the Caputo derivative which is compatible with the usual initial conditions.
Fractional calculus has been developed by many great mathematicians such as N. H. Abel, G. W. Leibniz, D. S. Laplace, L. Euler, J. Fourier, A. K. Grünwald, J. Hadamard, G. H. Hardy, O. Heaviside, A. V. Letnikov, J. Liouville, B. Riemann, M. Riesz, H. Weyl, A. Erdelyi, H. Kober, H. Holmgren, A. Zygmund, D. V. Willer,... and now there are hundreds of papers in the subject and several monographs and books.
The first book entirely devoted to the subject is the book by Oldham and Spanier [6]. Other books are: Samko, Kilbas and Marichev [40], Podlubny [7], Kilbas, Strivastava and Trujillo [39], Mainardi [4].
There are also several journals in the subject: like "Journal of Fractional Calculus", "Fractional Calculus and Applied Analysis"and the recent one "Fractional Dynamic Systems".
The interest to fractional calculus has been accelerated the past three decades after the publication of the three papers of Bagley and Torvik [1, 2, 3] and the paper by Podlubny [8]. The authors proved by experiments that when using fractional derivatives for viscoelastic materials

- experimental data (amplitude/frequency and dispersions) are in agreement within a broad frequency range
- time domain responses as stress relaxation and creep are well represented,
- the number of parameters is significantly reduced and
- the fractional model has some other features.

In addition to that, Podlubny established in [8] a geometric interpretation of the fractional integral and a physical interpretation of the fractional derivative.

Since then fractional calculus has attracted a large number of researchers and became very popular in numerous fields of sciences and engineering. It has been shown that fractional derivatives and fractional integrals are very successful in describing for instance anomalous kinetics and continuous time random walks. In general it has been confirmed that fractional derivatives are more suitable and adequate than derivatives of integer orders for the description of properties of many materials (especially materials with memory) and hereditary phenomena and processes. There are many other complex systems such as atmospheric diffusion of pollution, network traffic, anomalous diffusion, chaotic processes, biology, medicine, modelling and identification, electronics and wave propagation, mechanics, astrophysics, signal processing, chaotic dynamics, optics, powers media, etc.

Recently, new fractional derivatives which interpolate the Riemann–Liouville, Caputo, Hilfer, generalized fractional derivatives have appeared, see [39, 41, 43, 50, 51, 52]. Some investigators have recognized

that innovation for novel fractional derivatives with various nonsingular or singular kernels is necessary to address the need to model more realistic problems in various areas of engineering and science. Caputo and Fabrizio, in [44] introduced a new kind of fractional derivatives where the kernel based on the exponential function. Losada and Nieto in [53] studied some properties of this new operator. In [45, 46] the authors presented new interesting fractional derivatives where the kernel relies on the Mittag-Leffler function and generalized Mittag-Liffler function, which are basically generalizations of the Caputo fractional derivative. Then in [54, 55], the authors deliberated the discrete versions of those new operators. There are many works pertinent to ABC problem in medical science and engineering. Hence we highlight medical applications as well as engineering applications.

1.2 Theory of thermoelasticity

Deformation of a body is connected with the change of heat content and therefore with a change of temperature distribution of the body. A deformation of a body that varies in time leads to temperature changes and conversely. Thermoelasticity is the study of thermodynamical changes in bodies in equilibrium, whose interactions are limited to mechanical work, heat exchange, and external work. The classical dynamic elasticity was developed under the simplified assumption that heat exchange between the parts of the body due to heat conduction occurs slowly and hence motion may be regarded as adiabatic. Thermoelasticity deals with the theory of heat conduction, the theory of thermal stresses, temperature distribution produced by deformation, and thermoelastic dissipation.

In (1956) Biot [10] subedited the theory of coupled thermoelasticity to remove the paradox inveterate in the traditional uncoupled theory that flexible changes have no effect on the temperature. The heat equations for the two theories are of the diffusion kind anticipating infinite velocities of proliferation for heat waves contrary to physical observations. In (1967), Lord and Shulman [11] presented the theory of generalized thermoelasticity with one relaxation time for the special case of the anisotropic body.

This theory has been extended to include the anisotropic case by Dhaliwal and Sherief (1980) in [12]. In this theory, a modified law of thermal conductivity that includes both heat flow and its time derivative is replaced in place of the traditional Fourier law. The heat equation associated with this theory is hyperbolic and thus eliminates the paradox of infinite propagation velocities inherent in both discrete and double thermoelastic theories.

The qualitative properties of solution for a theory of thermoelasticity have been discussed under various conditions by many authors, here we refer to [13, 14, 15, 16]. The state space approach to this theory was developed by Anwar and Sherief (1988) and Sherief (1993) in [17, 18].

The essential solution for this theory has been obtained by Sherief (1986) [20]. Sherief and Hamza (1994, 1996) [21, 22] have solved some two-dimensional problems. In (2005) [24] Sherief et al., extended this theory to deal with micropolar materials. Sherief et al. (2004, 2005) in [23, 25] have developed the theory of thermoelastic diffusion. A new model of thermoelasticity is derived using the approach of fractional calculus and proving uniqueness theorem for this model by Sherief et al., in [19].

Recently, Yu and Zhao [56] simplified the theoretical frameworks by clarifying connections between existed models. Based on Laplace transform, the analytical solution for coupled thermoelasticity analysis is presented by Hosseini [57]. The theory of a modified fractional order generalized bio thermoelasticity with variable thermal material properties is developed by [?].

1.3 Some useful definitions

1. Riemann-Liouville fractional integral [39]

$$I_{a+}^{\alpha} y(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t-s)^{\alpha-1} y(s) ds.$$

2. Riemann-Liouville fractional derivative [39]

$$\begin{aligned} {}^{RL}D_{a+}^{\alpha} y(t) &= D^n \, I_{a+}^{n-\alpha} y(t), \quad D^n = \left(\frac{d}{dt}\right)^n \\ &= \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt}\right)^n \int_a^t (t-s)^{n-\alpha-1} y(s) ds. \end{aligned}$$

3. Caputo fractional derivative [39]

$$\begin{aligned} {}^{C}D_{a+}^{\alpha} y(t) &= I_{a+}^{n-\alpha} \, D^n \, y(t) \\ &= \frac{1}{\Gamma(n-\alpha)} \int_a^t (t-s)^{n-\alpha-1} y^{(n)}(s) ds. \end{aligned}$$

4. Hilfer fractional derivative [42]

$$ {}^{H}D_{a^+}^{\alpha,\beta}y(t) = I_{a^+}^{\beta(n-\alpha)}\left(\frac{d}{dt}\right)^n I_{a^+}^{(1-\beta)(n-\alpha)}y(t). $$

5. Generalized Riemann-Liouville fractional integral [39, 40]

$$ I_{a^+}^{\alpha;\psi}y(t) = \frac{1}{\Gamma(\alpha)}\int_a^t \psi'(\tau)(\psi(t)-\psi(\tau))^{\alpha-1}g(\tau)d\tau. $$

6. Generalized Riemann-Liouville fractional derivative [39, 40]

$$ {}^{RL}D_{a^+}^{\alpha;\psi}y(t) = \left(\frac{1}{\psi'(t)}\frac{d}{dt}\right)^n I_{a^+}^{n-\alpha;\psi}y(t). $$

7. Generalized Caputo fractional derivative [41]

$$ {}^{C}D_{a^+}^{\alpha;\psi}y(t) = I_{a^+}^{n-\alpha;\psi}\left(\frac{1}{\psi'(t)}\frac{d}{dt}\right)^n y(t). $$

8. Generalized Hilfer fractional derivative [43]

$$ {}^{H}D_{a^+}^{\alpha,\beta,\psi}y(t) = I_{a^+}^{\beta(n-\alpha),\psi}\left(\frac{1}{\psi'(t)}\frac{d}{dt}\right)^n I_{a^+}^{(1-\beta)(n-\alpha),\psi}y(t). $$

9. Caputo-Fabrizio fractional integral [44]

$$ {}^{CF}I_{a^+}^{\alpha}y(t) = \frac{2(1-\alpha)}{(2-\alpha)\mathcal{N}(\alpha)}y(t) + \frac{2\alpha}{(2-\alpha)\mathcal{N}(\alpha)}\int_a^t y(s)ds. $$

10. Caputo-Fabrizio fractional derivative [44]

$$ {}^{CF}D_{a^+}^{\alpha}y(t) = \frac{(2-\alpha)\mathcal{N}(\alpha)}{2(1-\alpha)}\int_a^t exp\left(\frac{-\alpha}{1-\alpha}(t-s)\right)y'(s)ds. $$

11. Atangana-Baleanu fractional integral [45]

$$ {}^{AB}I_{a^+}^{\alpha}y(t) = \frac{1-\alpha}{\mathcal{N}(\alpha)}y(t) + \frac{\alpha}{\mathcal{N}(\alpha)}\frac{1}{\Gamma(\alpha)}\int_a^t (t-\theta)^{\alpha-1}y(s)ds. $$

12. Atangana-Baleanu-Caputo fractional derivative [45]

$$ {}^{ABC}D_{a^+}^{\alpha}y(t) = \frac{\mathcal{N}(\alpha)}{1-\alpha}\int_0^t \mathbb{E}_{\alpha}\left(\frac{-\alpha}{\alpha-1}(t-s)^{\alpha}\right)y'(s)ds. $$

13. Tempered-Caputo fractional derivative [47]

$$ {}^{TC}D_{a^+}^{\alpha} y(t) = e^{-\delta t}\ {}^{C}D_{a^+}^{\alpha;\psi}\left[e^{\delta t} y(t)\right]. $$

14. Conformable fractional derivatives [48, 49]

$$ T_\alpha y(t) = \lim_{\epsilon \longrightarrow 0} \frac{y(t + \epsilon t^{1-\alpha}) - y(t)}{\epsilon}, \quad 0 < \alpha < 1. $$

where $n = [\alpha] + 1$ and $[\alpha]$ denotes the integer part of the real number α, $0 \leq \beta \leq 1$, ψ be an increasing function such that $\psi'(t) \neq 0$, $\mathcal{N}(\alpha)$ is the normalization function which is defined as $\mathcal{N}(0) = \mathcal{N}(1) = 1$, and $\mathbb{E}_\alpha$ is called the Mittag-Leffler function defined by the series

$$ E_\alpha(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + 1)}. $$

2 Literature Review

Several generalized heat conductive models have been introduced, of which the most celebrated models include e.g.

- CV model provided by Cattaneo and Vernotte [26, 27]:

$$ q_i + \tau \dot{q}_i = -kT_{,i}. \tag{1} $$

- DPL model provided by Tzou [28]

$$ q_i + \tau \dot{q}_i = -k\left(T_i + \tau_\theta \dot{T}_{,i}\right). \tag{2} $$

- GN model (Type-two) provided by Green and Naghdi [29, 30]

$$ \dot{q}_i = -k^* T_{,i}. \tag{3} $$

- GN model (Type-three) provided by Green and Naghdi [29]

$$ \dot{q}_i = -\left(kT_{,i} + k^* T_{,i}\right). \tag{4} $$

The last several years have attestation dramatically increasing demands on the employment of fractional calculus for describing complex phenomena. In this regards, the fractional derivative has been widely adopted in thermoelasticity, for instance:

- Povstenko model [31]:

$$ q_i = -k\ {}^{RL}D^{1-\alpha} T_{,i}, \quad 0 < \alpha \leq 1, \tag{5} $$

where ${}^{RL}D^{1-\alpha}$ denotes Riemann-Liouville fractional derivative of order $1-\alpha$. Based upon Fourier's law.

- Sherief model [19]:

$$q_i + \tau\, {}^{C}D^{\alpha} q_i \;= -kT_{,i}, \quad 0 < \alpha \leq 1, \tag{6}$$

where ${}^{C}D^{\alpha}$ denotes Caputo fractional derivative of order α. Based upon CV heat conductive law.

- Ezzat model [32, 33]:

$$q_i + \frac{\tau^{\alpha}}{\alpha!}\, {}^{C}D^{\alpha} q_i \;= -kT_{,i}, \quad 0 < \alpha \leq 1, \tag{7}$$

where ${}^{C}D^{\alpha}$ denotes Caputo fractional derivative of order α. Based upon CV heat conduction. This model has introduced significant applications in thermoelastic analyses.

- Youssef model [34, 35, 36]:

$$q_i + \tau \dot{q}_i \;= -kI^{\alpha-1}T_{,i}, \quad 0 < \alpha \leq 2, \tag{8}$$

where $I^{\alpha-1}$ denotes Riemann-Liouville fractional integral of order $\alpha - 1$. This generalized model is another extension of the CV model (Youssef's fractional heat conduction) with fractional calculus, which was used for analyses of thermoelastic responses.

- Abbas [37] proposed a fractional model based upon the second-type GN thermoelastic theory by adopting the same approach of Youssef's model, as

$$\dot{q}_i \;= -k^{*}I^{\alpha-1}T_{,i}, \quad 0 < \alpha \leq 2. \tag{9}$$

- Recently, Li et al. [38] have formulated the fractional GN model (9) by introducing relaxation time as follows:

$$\tau^{\alpha-1}\, {}^{C}D^{\alpha} q_i = -k^{*}T_{,i}, \quad 0 < \alpha < 1. \tag{10}$$

- In (2020) Yu, Li Jing Zhao [56] have proposed the generalized Fourier's law Inspired by the progress made for the fractional GN model (Li et al. [?], 2018), as follows:

$$q_i = -k\tau^{1-\alpha}\, D_m^{1-\alpha}\theta_{,i}, \quad 0 < \alpha \leq 1, \tag{11}$$

where $\theta_{,i} = T - T_0$ is temperature increment with T_0 the reference temperature, and $D_m^{1-\alpha}$ denotes different definitions fractional derivative of order $1-\alpha$.

2.1 Our intention

On the other hand, interesting advances have been made in the fractional calculus by proposed some new forms of fractional derivative, e.g., ψ-Riemann-Liouville-type [39], ψ-Caputo-type ([41],2017), ψ-Hilfer-type ([43], 2018), Caputo-Fabrizio-type ([44], 2015), Atangana–Baleanu- type ([45], 2016), Tempered-Caputo-type fractional derivatives ([47], 2019). It is reasonable to believe that various fractional derivative definitions predict different responses. By the aids of such new fractional derivatives, the present proposal is aimed to

1. Construct a generalized fractional thermoelasticity.
2. New modified models of generalized thermoelasticity will be derived based on fractional calculus.
3. The proposed models will be a direct extension of Fourier's law.
4. The proposed models are expected to be able to generalization Sherief, Ezzat, and Youssef's models.
5. Formulate and solve mathematical models of different fractional thermoelasticity problems.
6. Publication of the research findings as research papers / articles in journals.
7. To compile the research work in the form of Ph. D thesis.

3 Problems Statement

We will study the generalized thermoelasticity involving advanced fractional calculus:

$$q_i = -k\left(\psi(\tau) - \psi(0)\right)^{1-\alpha} \; D_m^{1-\alpha,\psi}\theta_{,i}, \quad 0 < \alpha \leq 1, \tag{12}$$

where $\theta_{,i} = T - T_0$ is temperature increment with T_0 the reference temperature, and $D_m^{1-\alpha,\psi}$ denotes different definitions generalized fractional derivative of order $1-\alpha$.

Note that,

(i) If $m = 1$, and $\psi(t) = t$, the problem (12) reduce to the problem (11) see [56].

(ii) If $m = 2$, and $\psi(t) = t$, the problem (12) reduce to the Caputo-Fabrizio-type problem

$$q_i = -k\tau^{1-\alpha \; CF}D_m^{1-\alpha}\theta_{,i}, \quad 0 < \alpha \leq 1. \tag{13}$$

(iii) If $m = 3$, and $\psi(t) = t$, the problem (12) reduce to the Atangana-Baleanu-Caputo-type problem

$$q_i = -k\tau^{1-\alpha \; ABC}D_m^{1-\alpha}\theta_{,i}, \quad 0 < \alpha \leq 1. \tag{14}$$

(iv) If $m = 4$, and $\psi(t) = t$, the problem (12) reduce to the Tempered-Caputo-type problem

$$q_i = -k\tau^{1-\alpha}\, {}^{TC}D_m^{1-\alpha}\theta_{,i}, \quad 0 < \alpha \leq 1. \tag{15}$$

(v) If $m = 1$, and the function $\psi(t)$ takes different values, then our problem (12) covers many problems related to modern fractional operators. For instance:

$$\psi(t) = \log(t),\ D_1^{1-\alpha,\psi} \rightarrow D_1^{1-\alpha,\log}(\text{Hadamard fractional derivative}),$$

$$\psi(t) = t^{\rho}\ (\rho > 0),\ D_1^{1-\alpha,\psi} \rightarrow D_1^{1-\alpha,t^{\rho}}(\text{Katugampola fractional derivative}).$$

4 Motivation and Objectives

In order to fill the gaps in the literature, the following objectives of the present study have been laid:

1. To propose novel modified models of generalized thermoelasticity involving fractional calculus.
2. To apply the state-space approach, Laplace transformation, and Fourier transformation to solve some problems of fractional generalized thermoelasticity.
3. To prove the uniqueness theorems for these models.
4. To analyze and derive the dynamic problems and the governing equations of fractional coupled thermoelasticity based on integral transformations.
5. To develop the Povstenko model (2011), Green-Nigdhi (GN) Model (1993), and Sherief model (2010) in the frame of modern fractional calculus.

5 Need and Significance

The fractional calculus deals with the generalization of differentiation and integration to arbitrary order. Many phenomena in various fields of science and engineering can be described by differential equations of non-integer order. Namely, they arise naturally in viscoelasticity, porous media, electrochemistry, control, electromagnetic, etc. [6, 7, 39].
In fact it has been shown by experiments that derivatives of non-integer order can describe many phenomena better than derivatives of integer order specially hereditary phenomena and processes.
Fractional derivatives by definition involve all the history of the state through a convolution with a singular kernel. In addition to this singularity, the convolution term is non-local in time. This fact complicates considerably the use of the existing methods in the literature.

Using the direct integration method, they found continuous and integral solutions of differential equations of quasi-planar thermal elasticity with boundary conditions of

mixed type, displacement or traction. They have also established individual relationships between tracking operations and cross-border displacement. To ensure correct solutions to thermal elasticity problems, integrative equilibrium conditions necessary for traction, conditions for full compatibility of displacement processes and thermal equilibrium conditions were formulated. Due tothe great importance of thermal elasticity and its many applications in practical life, and its connection to some other important sciences such as materials engineering and nanotechnology, and for these reasons we are doing this study which we are interested in studying the link between thermal elasticity and materials engineering On the one hand, and on the other hand, its relationship to nanotechnology. To this end, we study differential equations when their order is not an integer because the ordinary differential equations no longer satisfy the solution of the problems that resulted from the current association between thermal elasticity and these new sciences.

6 Methodology

This research is based on the descriptive approach. Secondary data is used through the use of scientific research, books, dissertations, reports, journals and articles.

The generalized thermoelasticity involving advanced fractional calculus for the problems or models we will consider should not be difficult. In fact, there are many results in this regard and we may find appropriate over for our cases. Otherwise, we will be forced to establish such results.

We plan to achieve our objectives through

(I) Considering the fractional generalized thermoelasticity.

(II) Applying the state-space approach, Laplace transformation, and Fourier transformation.

(III) Applying the Povstenko model, Green-Nigdhi (GN) Model, and Sherief model in the frame of modern fractional calculus.

7 Proposed Work plan /Formulation and Structure of study

Sr.No	Period	Plan of the work	Targets to be achieved
1	First year	Study of the background required for the research topic, such as fractional calculus and thermoelasticity	Good command on the subject and clear the course work
2	Second year	Collecting the papers and analyze them	Try to get some new result based on the previous work
3	Third year	Work on some problems to be included in the thesis	Submit at least two papers
4	Fourth year	Publishing some papers and submitting the Thesis	Some more papers and Thesis submission

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